

236646-נושאים מתקדמים בתאוריה של מדעי המחשב ה

הסיבוכיות של טרנספורציית פורייה ובעיות נוספות

סמסטר חורף תשע"ט

מרצה – פרופ'ח ניר אילון

Complexity of Fourier Transform and Related Problems

The Fourier Transform is one of the most important linear transformations used in science and engineering, with CS applications (to name just a few) spanning fast integer multiplication, machine learning and dimensionality reduction.

Cooley and Tukey's Fast Fourier Transform (FFT) [1] is a method for computing this transformation in time $O(n \log n)$. In spite of its importance, however, from a lower bound perspective, relatively little is known optimality of FFT. The most highly cited relevant results are by Morgenstern [2] and by Papadimitriou [3], but they are quite restrictive, and not satisfactory for various reasons that will be explained in class.

Recently, there has been progress in understanding the complexity of the Fourier transform [4,5], which leads to more interesting open problems.

The course will cover:

1. Definition and applications of Fourier transform, the Fast Fourier Transform
2. Lower Bounds - Old and recent results
3. Related and tangential topics such as sparse Fourier transform [6] and quantum Fourier transform.

The goal of the course is to introduce students to interesting research topics, and the coursework will support this goal. Students will be asked to read papers related to a specific problem, and summarize them or present them.

[1] James W. Cooley and John W. Tukey. An algorithm for the machine calculation of complex Fourier series. *Mathematics of Computation*, 19:297–301, 1965.

[2] Jacques Morgenstern. Note on a lower bound on the linear complexity of the fast Fourier transform. *J. ACM*, 20(2):305–306, April 1973.

[3] Christos H. Papadimitriou. Optimality of the fast Fourier transform. *J. ACM*, 26(1):95–102, January 1979

[4] Nir Ailon. A lower bound for Fourier transform computation in a linear model over 2×2 unitary gates using matrix entropy. *Chicago J. of Theo. Comp. Sci.* 2013.

[5] Nir Ailon. Tighter Fourier transform lower bounds. In *ICALP 2015*.

[6] Piotr Indyk and Michael Kapralov and Eric Price. (Nearly) sample-optimal sparse Fourier transform. *SODA 2014*