Graph polynomials by interpreting sequences of finite relational structures

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Abstract

The chromatic polynomial of a graph G evaluated at a positive integer n is equal to the number of homomorphisms from G to K_n, the complete graph on n vertices. Many important graph polynomials are likewise determined by counting homomorphisms to a sequence of graphs, among which are the Tutte polynomial and the independence polynomial.

De la Harpe and Jaeger introduced the notion of a *strongly polynomial sequence* of graphs for a sequence of (G_n) of finite graphs sharing this property that, for every graph G, the number of homomorphisms from G to G_n is a fixed polynomial function of n (depending on G).

In this earlier work of de la Harpe and Jaeger, and more recently of Averbouch, Garijo, Godlin, Goodall, Kotek, Makowsky, Nešetřil, Tittmann, Zilber and others, various examples of strongly polynomial sequences and constructions for families of such sequences have been found, leading to analogues of the chromatic polynomial for fractional colourings and acyclic colourings, to choose two interesting examples.

I will sketch a model-theoretic method of constructing strongly polynomial sequences of graphs that uses interpretation schemes of finite graphs in more general finite relational structures. This surprisingly easy yet general method encompasses all previous constructions and produces many more. We conjecture that, under mild assumptions, all strongly polynomial sequences of graphs can be produced by quantifier-free interpretation of graphs in certain basic finite relational structures (essentially disjoint unions of transitive tournaments with added unary relations).

Reference A.J. Goodall, J. Nešetřil and P. Ossona de Mendez, Strongly polynomial sequences as interpretations of trivial structures, J. Appl. Logic 18 (2016), 129–149. doi:10.1016/j.jal.2016.06.001. Preprint at arXiv:1405.2449 [math.CO].